that exchanges between the active and stagnant regions

Pe = Peclet number

Re = liquid Reynolds number

V = volume; subscripts: p-dispersed film, m-mixing tank, s-stagnant zone, t-total

θ = dimensionless time, based on the mean holding time of the total sequence

LITERATURE CITED

Bennett, A., and F. Goodridge, "Hydrodynamics and Mass Transfer Studies in Packed Absorption Columns," *Trans. Inst. Chem. Engrs.*, **48**, 232 (1970).

Buffham, B. A., L. G. Gibilaro, and M. N. Rathor, "A Probabilistic Time Delay Description of Flow in Packed Beds," *AIChE J.*, **16**, 218 (1970).

Furzer, I. A., and R. W. Michell, "Liquid Phase Dispersion in Packed Beds with Two-Phase Flow," *ibid.*, 380 (1970).

Hoogendoorn, C. J., and J. Lips, "Axial Mixing of Liquid in Gas-Liquid Flow through Packed Beds," *Can. J. Chem. Eng.*, **43**, 125 (1965).

Levenspiel, O., and W. K. Smith, "Notes on the Diffusion Type Model for the Longitudinal Mixing of Fluids in Flow," *Chem. Eng. Sci.*, **6**, 227 (1956).

Levich, V. G., V. S. Markin, and Yu. A. Chismadzhev, "On Hydrodynamic Mixing in a Model of a Porous Medium with Stagnant Zones," *ibid.*, **22**, 1357 (1967).

Michell, R. W., and I. A. Furzer, "Trickle Flow in Packed Beds," *Trans. Inst. Chem. Engrs.*, **50**, 334 (1972).

Otake, T., and E. Kunigita, "Mixing Characteristics of Irrigated Packed Towers," *Kagaku Kogaku*, **22**, 144 (1958).

Pham Co, and R. Ribaud, "Longitudinal Mixing of the Liquid Phase in Packed Columns with Countercurrent Two-Phase Flow," *Can. J. Chem. Eng.*, **49**, 727 (1971).

Sater, V. E., and O. Levenspiel, "Two-Phase Flow in Packed Beds," *Ind. Eng. Chem. Fundamentals*, **5**, 86 (1966).

Satterfield, C. N., "Trickle-Bed Reactors," *AIChE J.*, **21**, 209 (1975).

Schwartz, J. G., and G. W. Roberts, "An Evaluation of Models for Liquid Backmixing in Trickle-Bed Reactors," *Ind. Eng. Chem. Process Design Develop.*, **12**, 262 (1973).

van Swaaij, W. P. M., J. C. Charpentier, and J. Villermaux, "Residence Time Distribution in the Liquid Phase of Trickle Flow in Packed Columns," *Chem. Eng. Sci.*, **24**, 1083 (1969).

Vasudeva, M., M. Tech. thesis, I. I. T. Madras, India (1974).

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Drag Reduction in Two-Phase Annular-Mist Flow of Air and Water

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Two-phase, gas-liquid flow in pipes is frequently encountered and is of significant commercial importance in the petroleum and natural gas industries. The complexity of two-phase momentum transfer problems has led mainly to an accumulation of experimental data and to the analysis of the data by semiempirical extension of single-phase flow models (Govier and Aziz, 1972). The literature in the field is extensive; for example, a general index of 5,232 references on two-phase flow has been compiled by Gouse (1966).

Turbulent flow-drag reduction can be defined as a decrease in the pressure gradient for a given volumetric flow rate. It is usually produced by the addition of a linear, high molecular weight, polymeric material to a low viscosity liquid. Drag reduction has received considerable attention because of the many applications of both theoretical and pragmatic interest. The numerous published studies are almost exclusively concerned with single-phase liquid flow. A number of these single-phase studies are contained in symposium volumes edited by Wells (1969), Savins and Virk (1971), and Sylvester (1973). In addition, excellent reviews of single-phase drag reduction have been published by Patterson et al. (1969), Hoyt (1972), Lumley (1973), and Virk (1975). The mechanism responsible for drag reduction with polymer solutions has not been satisfactorily explained (see, for example, Kumor and Sylvester, 1973; Virk, 1975). The mechanism remains obscure partly because of experimental difficulties (Virk, 1975) and polymer degradation during flow (Sylvester and Kumor, 1973).

Although there have been numeorus studies of drag reduction in single-phase pipe flow of polymer solutions, only four studies of two-phase, gas-liquid, horizontal pipe flow with polymers dissolved in the liquid phase have been published (Oliver and Hoon, 1968; Mahalingam and Valle, 1972; Greskovich and Shrier, 1971; Rosehart et al., 1972). Of these four, all of which were low pressure studies, only two (Greskovich and Shrier, 1971; Rosehart et al., 1972) were concerned with drag reduction. Their experimental work was limited to relatively low gas-liquid ratios (plug or slug flow) not likely to be found in gas production and transportation.

There are a number of potential applications of drag reduction in two-phase flow to gas production and transportation. Examples include the reduction of pumping and compressor requirements in gas-gas condensate pipelines, increased capacity of existing producing equipment, and increased flow from aquifers and other storage facilities.

The purpose of this study was to demonstrate the existence of drag reduction in two-phase, annular-mist flow. Pressure drop data were taken for horizontal flow of air and water in a 1.27 cm diameter pipe at a system pressure of approximately 6.895×10^5 Pa (100 lb/sq in. gauge). Polyethylene oxide, a well-known, water soluble, drag reducing polymer was used. The liquid-gas ratio was varied from 56.2 to 5620 m^3 of liquid per million standard cubic meter of gas (that is, 10 to 1000 barrels of liquid per million standard cubic feet of gas).

EXPERIMENT

The experimental equipment is shown schematically in Figure 1. The air supply was provided by two Ingersoll-Rand compressors and maintained at 9.31×10^5 Pa (135 lb/sq. in. gauge). The air feed line contained an orifice meter run and a pressure regulator which maintained the line pressure at 8.69 $\times$ 10^5 Pa (126 lb/sq in. gauge). The orifice meter monitored the air temperature, the absolute pressure, and the differential pressure drop to provide an accurate and continuous air flow measurement. Water was fed to the system by a FWI triplex pump driven by a variable speed motor. All tubing following the mixing tee was 1.27 cm I.D. stainless steel and was mounted in the same horizontal plane. A calming section of

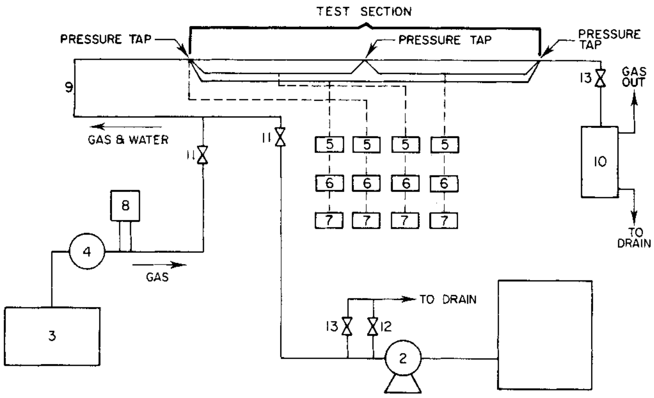

Fig. 1. Schematic diagram of experimental equipment (numbers refer to Table 1).

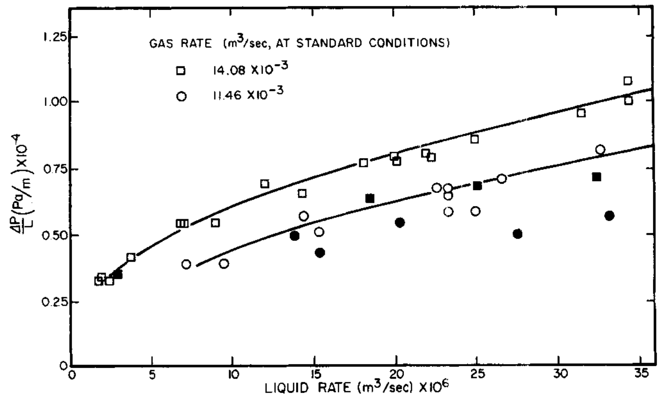

Fig. 2. Two-phase pressure gradient vs liquid flow rate.

TABLE 1

1. Liquid feed tank
2. Liquid feed pump
3. Gas supply
4. Pressure regulator
5. Validyne pressure transducers
6. Validyne carrier-demodulators
7. Esterline-Angus strip chart recorders
8. Orifice meter (1.905 cm Daniel)
9. Piping (1.27 cm I.D., 304 stainless steel)
10. Horizontal gas-liquid separator
11. Check valves
12. Bypass safety valve
13. Needle valves

approximately 1,100 tube diameters was provided between the mixing tee and the test section. The test section was 6.096 m long and contained three pressure taps dividing it into two 3.048 m sections. The exiting air-water mixture was passed to a gas-liquid separator at 5.93×10^5 Pa (86 lb./sq. in. gauge). The air was vented and the liquids wasted. The three pressure taps were connected to four Validyne pressure transducers. The pressure transducers provided direct measurement of the pressure at the beginning of the test section, the differential pressure drops across the two 3.048 m sections, and the pressure drop across the entire 6.096 m section. The transducers were wired to individual Validyne demodulators and their outputs recorded on Esterline-Angus Strip chart recorders.

An experimental run was started by passing a given flow rate of dry air through the system to check the consistency of the three differential pressure transducers which were calibrated before every run. Water at a given flow rate was then admitted to the system and all temperature, pressure drop, and flow rate measurements made. The air flow rate was varied to provide a data set. The water flow rate was changed and the above procedure repeated to generate another data set.

The polymer solution data were obtained in a similar fashion. At time zero, approximately 1 gal. of a concentrated (5 000 wppm) polymer solution was added to approximately 50 gal of water in the feed tank. The feed tank mixer provided rapid mixing to a concentration of 100 wppm, and this solution was continuously fed to the system.

The concentrated polymer solutions were prepared by adding polymer to water with gentle mixing and by allowing the mixture to stand overnight. The viscometric properties of the polymer solutions were measured at 25°C with a Ubbelohde dilution viscometer. The polymer used was polyethylene oxide (Polyox-FRA) provided by Union Carbide, Inc.

RESULTS AND DISCUSSION

The experimental equipment and instrumentation were evaluated by measuring pressure drop data for air only. Friction factors calculated from pressure drop measurements were compared to those obtained from an equation for smooth pipes [that is, $f = 0.0014 + 0.125\ (N_{Re}^{-0.32})$]. The comparison was good with an average deviation of 4%.

Since no experimental data existed at the very high gas-liquid ratios to be studied, 57 experiments were performed for the air-water system covering a liquid-gas ratio range of from 56.2 to 5 620 m^3 of liquid per million standard cubic meter of gas. All the data were in the annular-mist flow regime. The measured pressure gradients were compared to those calculated using the accepted correlation (Baker et al., 1970). The measured gradients were always greater than the calculated gradients, and the percent difference ranged from 1 to 43%. These results are not unexpected in light of the limitations of the correlation.

The viscometric properties of the polymer solution studied were determined by standard methods. The intrinsic viscosity was found to be 8.9 deciliters/g, which compares favorably with previous measurements for similar polymer systems (Sylvester and Tyler, 1970).

Figure 2 is a plot of the experimental pressure gradient vs. the liquid flow rate for two average gas rates. The open squares and circles are air-water data points, and the shaded squares and circles are air-polymer solution data points. It is clear that significant pressure gradient reduction was obtained, and that it increased with increasing liquid rate at a fixed gas rate. Pressure gradient reductions up to 37% were obtained. The existence of significant drag reduction for two-phase, annular-mist flow has been demonstrated.

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NOTATION

D = pipe diameter

$\frac{\Delta p}{L}$ = pressure gradient

$f = \frac{\left(\frac{\Delta p}{L}\right) D}{2\rho v^2}$ = friction factor

$N_{Re} = \frac{\rho v D}{\mu}$ = Reynolds number

Q_L = liquid flow rate

Q_g = air flow rate

v = air velocity

ρ = air density

μ = air viscosity

LITERATURE CITED

Baker, O., H. W. Brainerd, C. L. Coldren, O. Flanigan, and J. K. Welchen, *Gas-Liquid Flow in Pipelines II. Design Manual*, AGA-API Manual prepared on Project NX-28, AGA Catalog No. L20269 (Oct., 1970).

Gouse, S. W., Jr., "An Index to the Two-Phase, Gas-Liquid Flow Literature," Mass. Inst. Technol. Reports DSR 8734-6 (1966).

Govier, G. W. and K. Aziz, *The Flow of Complex Mixtures in Pipes*, Van Nostrand Remhold, New York (1972).

Greskovich, E. J., and A. L. Shrier, "Drag Reduction in Two-Phase Flows," *Ind. Eng. Chem. Fundamentals*, **10**, 646 (1971).

Hoyt, J. W., "The Effect of Additives on Fluid Friction," *J. Basic Eng.*, **94D**, 258 (1972).

Kumor, S. M., and N. D. Sylvester, "Effects of a Drag-Reducing Polymer on the Turbulent Boundary Layer," in *Drag Reduction in Polymer Solutions*, N. D. Sylvester, ed., AIChE Symposium Series, Vol. 60, No. 130, pp. 1-13 (1973).

Lumley, J. L., "Drag Reduction in Turbulent Flow by Polymer Additives," *Macromol. Rev.*, 7, 263 (1973).

Mahalingam, R., and M. A. Valle, "Momentum Transfer in Two-Phase Flow of Gas-Pseudoplastic Liquid Mixtures," *Ind. Eng. Chem. Fundamentals*, **11**, 470 (1972).

Oliver, D. R., and A. Y. Hoon, "Two-Phase Non-Newtonian Flow," *Trans. Inst. Chem. Engrs.*, **46**, T106 (1968).

Patterson, G. K., J. L. Zakin, and J. M. Rodriguez, "Drag Reduction: Polymer Solutions, Soap Solutions, and Solid Particle Suspensions in Pipe Flow," *Ind. Eng. Chem.*, **61**, 22 (1969).

Rosehart, R. G., D. S. Scott, and E. Rhodes, "Gas-Liquid Slug Flow with Drag-Reducing Polymer Solutions," *AIChE.*, **18**, 744 (1972).

Savins, J. G., and P. S. Virk, ed., *Drag Reduction*, AIChE Symposium Series, Vol. 67, No. 111 (1971).

Sylvester, N. D., ed., *Drag Reduction in Polymer Solutions*, AIChE Symposium Series, Vol. 69, No. 130 (1973).

———, and S. M. Kumor, "Degradation of Dilute Polymer Solutions in Turbulent Tube Flow," in *Drag Reduction in Polymer Solutions*, N. D. Sylvester, ed., AIChE Symposium Series, Vol. 69, No. 130, pp. 69-81 (1973).

———, and J. S. Tyler, "Dilute Solution Properties of Drag-Reducing Polymers," *Ind. Eng. Chem. Product Research Develop.*, **9**, 548 (1970).

Virk, P. S., "Drag Reduction Fundamentals (Journal Review)," *AIChE J.*, **21**, 625 (1975).

Wells, C. S., ed., *Viscous Drag Reduction*, Plenum, New York (1969).

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Flow Behavior of Power Law Fluids in an Annulus

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Fredrickson and Bird (1958); Rotem (1962); McEachern (1966); Kozicki, Chou, and Tiu (1966); and Kozicki and Tiu (1971) have theoretically analyzed the fully developed laminar flow, and Tiu and Bhattacharya (1974) and Bhattacharya and Tiu (1974) have presented experimental results for developing and fully developed velocity and pressure profiles for inelastic power law fluids in an annulus. The pressure loss analysis of the entrance region flow and also the solution of energy equations require the knowledge of velocity profile in the developing boundary layer which is customarily chosen according to that in the fully developed region. Although El. Defrawi and Finlayson (1972) have pointed out the inappropriateness of the use of Newtonian velocity profile for non-Newtonian fluids in the boundary layer flow, Tiu and Bhattacharya (1973) had to opt for the second-order Newtonian profile for use in the momentum-energy integral technique, in view of the lengthy mathematics involved in the solution of the governing differential equations. Therefore, it seems desirable to obtain an approximate closed form of the velocity profile in fully developed flow satisfactorily agreeing with the exact analysis of Fredrickson and Bird (1958).

An annulus may be considered as the most general conduit of which a circular tube and parallel plate channel are two particular geometries. The local velocity profile for fluids, following a power law

$$\tau = K(-du/dr)^n \tag{1}$$

flowing through these two extreme shaped conduits may be expressed as

$$u/u_m = \left[1 - Z^{\frac{n+1}{n}}\right] \tag{2}$$

where Z is the dimensionless distance from zero shear position. On this basis the velocity profiles in the inner and outer flow regions of an annulus are proposed as

$$\frac{u_1}{u_m} = 1 - \left(\frac{r_m - r}{r_m - r_1}\right)^{\frac{n+1}{n}} ; \quad r_1 \leq r \leq r_m \tag{3}$$

and

$$\frac{u_2}{u_m} = 1 - \left(\frac{r - r_m}{r_2 - r_m}\right)^{\frac{n+1}{n}} ; \quad r_m \leq r \leq r_2 \tag{4}$$

From Equations (3) and (4), the average velocity in the annulus may be obtained as

$$\frac{\langle u \rangle}{u_m} = \left[\frac{(n+1)}{(1+k)}\right]\left[\frac{(2n+1)(1+k)+2n\lambda}{(3n+1)(2n+1)}\right] \tag{5}$$

By substituting the values of τ_{w1} and τ_{w2} obtained from the velocity gradients at the inner and outer walls, respectively, in the overall momentum balance equation, the maximum velocity may be expressed as

$$u_m = \left(\frac{\Delta P\, r_2}{2KL}\right)^{1/n} \left[\frac{r_2{}^n(1-k^2)}{(1-\lambda)^{-n} + k(\lambda - k)^{-n}}\right]^{1/n} \left(\frac{n}{n+1}\right) \tag{6}$$

From Equations (5) and (6), the volumetric flow rate Q may be expressed as

$$Q = \pi r_2{}^3 \left(\frac{\Delta P\, r_2}{2\,KL}\right)^{1/n} \Omega' \tag{7}$$